Efficiency of the detection in in-situ gamma spectrometry

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In in-situ gamma spectrometry we usually have the situation shown in Figure 1.

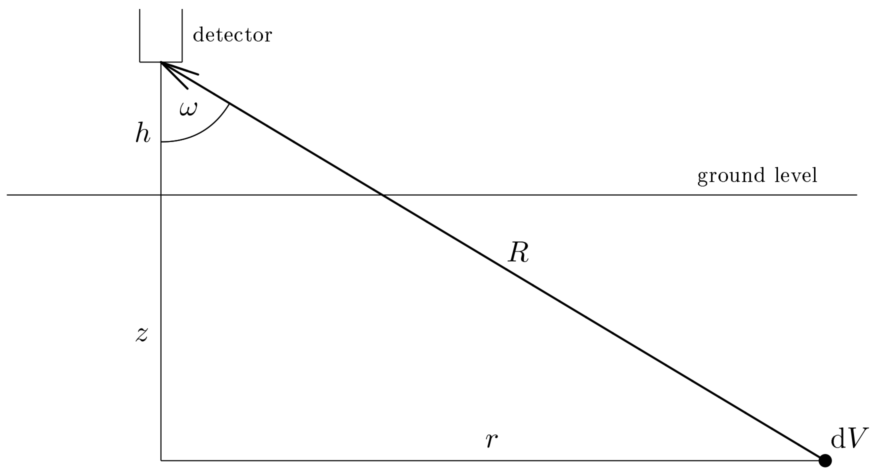

Figure 1. *A usual measurement setup in in-situ gamma spectrometry*

Here each dV volume element – according to the activity distribution in the ground – emits photons, which are detected to some extent by the detector. We need to know this extent, so that we can determine the activity concentration in the ground from the measurement results.

For this purpose let's calculate the count rate at a given activity distribution in the ground. How can we do that?

First of all let's calculate the count rate when all the photons are going into the same direction, described by the unit vector, $\mathbf{n}$. In this case we can introduce $\mathbf{j}(\mathbf{r})$, the photon current at place $\mathbf{r}$, which is the flux, $\phi(\mathbf{r})$, multiplied by $\mathbf{n}$. Further, let $\varepsilon(\mathbf{n},\mathbf{r})$ denote the probability that a photon, coming from direction $\mathbf{n}$ and hitting the detector at place $\mathbf{r}$, is detected. The count rate, Γ, can then be calculated as follows:

$$\Gamma = -\int_{A_{\mathbf{n}}} \varepsilon(\mathbf{n},\mathbf{r}) \cdot \mathbf{j}(\mathbf{r})\,\mathrm{d}\mathbf{a} = -\int_{A_{\mathbf{n}}} \varepsilon(\mathbf{n},\mathbf{r}) \cdot \phi(\mathbf{r}) \cdot \mathbf{n}\,\mathrm{d}\mathbf{a},$$

where $A_{\mathbf{n}}$ is the surface of the detector visible from direction $\mathbf{n}$. Note that here $\mathbf{j}(\mathbf{r})$ and $\mathrm{d}\mathbf{a}$ are vectors.

Provided that the photon flux is homogeneous over $A_{\mathbf{n}}$, which we then simply designate by ϕ, we can write that

$$\Gamma = -\phi \cdot \int_{A_{\mathbf{n}}} \varepsilon(\mathbf{n},\mathbf{r}) \cdot \mathbf{n}\,\mathrm{d}\mathbf{a}.$$

Introducing $\varepsilon(\mathbf{n})$ as

$$\varepsilon(\mathbf{n}) = -\int_{A_{\mathbf{n}}} \varepsilon(\mathbf{n},\mathbf{r}) \cdot \mathbf{n}\,\mathrm{d}\mathbf{a},$$

we can write

$$\Gamma = \phi \cdot \varepsilon(\mathbf{n}).$$

$\varepsilon(\mathbf{n})$ is the count rate at unit homogeneous flux from direction $\mathbf{n}$ – a characteristic of the detector, which can be determined by measurements. We can assume that the detector is symmetrical, that is its characteristics don't depend on the azimuthal angle, φ, but only on the polar angle, ω. Moreover, since $0 \le \omega \le \pi$, where the cos function is invertible, we can say that the efficiency actually depends on $\cos\omega$. Let ϵ denote this other efficiency function: $\varepsilon(\mathbf{n}) = \epsilon(\cos\omega)$.

We can use the approximation that all the photons coming from any single $\mathrm{d}V$ volume element in the ground go into the same direction at the detector surface and they provide a homogeneous flux over it. Thus we can use the above equations to calculate the count rate from each $\mathrm{d}V$ element. Then all we have to do is to integrate these "pieces of count rates".

Let A_V denote the activity concentration in the ground and let's suppose that A_V only depends on the depth, z. Further, let Y denote the yield and let μ_s and μ_a denote the attenuation coefficients in soil and air, respectively. μ_s may depend on the depth, z. Thus, the count rate can be calculated as follows:

$$\Gamma = \int \frac{A_V(z) \cdot Y}{4\pi R^2} \cdot e^{-\frac{1}{\cos\omega}\cdot\int_0^z \mu_s(z')\,\mathrm{d}z'} \cdot e^{-\mu_a \cdot \frac{h}{\cos\omega}} \cdot \epsilon(\cos\omega)\,\mathrm{d}V. \tag{1}$$

Here $\mathrm{d}V$ can be written as

$$\begin{aligned} \mathrm{d}V &= R^2 \cdot \sin\omega \,\mathrm{d}R \,\mathrm{d}\omega \,\mathrm{d}\varphi \\ &= R^2 \cdot \sin\omega \,\frac{\mathrm{d}z}{\cos\omega} \,\mathrm{d}\omega \,\mathrm{d}\varphi \\ &= -\frac{R^2}{\cos\omega} \,\mathrm{d}z \,\mathrm{d}(\cos\omega) \,\mathrm{d}\varphi. \end{aligned}$$

Now we can replace $\cos\omega$ with x. Since ω goes from 0 to $\pi/2$, x goes from 1 to 0 and the whole integral takes the form:

$$\begin{aligned} \Gamma &= \int_0^{2\pi}\!\!\int_0^1\!\!\int_0^\infty \frac{A_V(z)\cdot Y}{4\pi R^2} \cdot e^{-\frac{1}{x}\cdot\int_0^z \mu_s(z')\,\mathrm{d}z'} \cdot e^{-\mu_a\cdot\frac{h}{x}} \cdot \frac{\epsilon(x)}{x} \cdot R^2 \,\mathrm{d}z \,\mathrm{d}x \,\mathrm{d}\varphi \\ &= \frac{Y}{2} \cdot \int_0^1 \mathrm{d}x \,\frac{\epsilon(x)}{x} \cdot e^{-\mu_a\cdot\frac{h}{x}} \cdot \int_0^\infty \mathrm{d}z \,A_V(z) \cdot e^{-\frac{1}{x}\cdot\int_0^z \mu_s(z')\,\mathrm{d}z'}. \end{aligned} \tag{2}$$

Here the integration over φ has been done, which is actually a multiplication by 2π. In order to do the integration over z too, we need to know the activity distribution in the soil. In some special cases, with the assumption that μ_s doesn't depend on the depth, we can do this integration easily. The remaining integration over x has to be done numerically.

Exponential distribution
In this case $A_V(z) = A_0 \cdot e^{-\lambda\cdot z}$. Thus the count rate is:

$$\Gamma = \frac{A_0 \cdot Y}{2} \cdot \int_0^1 \frac{\epsilon(x)}{\lambda\cdot x + \mu_s} \cdot e^{-\mu_a\cdot\frac{h}{x}} \,\mathrm{d}x. \tag{3}$$

Homogeneous distribution
In this case $A_V(z) = A_0$. Thus the count rate becomes:

$$\Gamma = \frac{A_0 \cdot Y}{2\cdot\mu_s} \cdot \int_0^1 \epsilon(x) \cdot e^{-\mu_a\cdot\frac{h}{x}} \,\mathrm{d}x. \tag{4}$$

Homogeneous distribution down to depth p and nothing below it
In this case $A_V(z) = A_0$ if $0 \le z \le p$ and $A_V(z) = 0$ if $z > p$. Thus the count rate is:

$$\Gamma = \frac{A_0 \cdot Y}{2\cdot\mu_s} \cdot \int_0^1 \epsilon(x) \cdot e^{-\mu_a\cdot\frac{h}{x}} \cdot \left(1 - e^{-\mu_s\cdot\frac{p}{x}}\right) \,\mathrm{d}x. \tag{5}$$

Sometimes only the surface of the ground is contaminated with radioactive material. Supposing that this contamination is homogeneous, we can introduce A_S as the activity concentration on the surface. We can repeat the above train of thought to derive the following equation:

$$\begin{aligned}\Gamma &= \int \frac{A_S \cdot Y}{4\pi(h^2+r^2)} \cdot e^{-\mu_a \cdot \sqrt{h^2+r^2}} \cdot \epsilon(r)\,\mathrm{d}a \\ &= \int_0^{2\pi}\int_0^{\infty} \frac{A_S \cdot Y}{4\pi(h^2+r^2)} \cdot e^{-\mu_a \cdot \sqrt{h^2+r^2}} \cdot \epsilon(r) \cdot r\,\mathrm{d}r\,\mathrm{d}\varphi. \end{aligned} \tag{6}$$

(For the meaning of r see Figure 1.) Introducing $x := \frac{h}{\sqrt{h^2+r^2}}$, this equation can be written as

$$\begin{aligned}\Gamma &= \int_0^{2\pi}\int_0^{1} \frac{A_S \cdot Y}{4\pi x} \cdot e^{-\mu_a \cdot \frac{h}{x}} \cdot \epsilon(x)\,\mathrm{d}x\,\mathrm{d}\varphi \\ &= \frac{A_S \cdot Y}{2} \cdot \int_0^1 \frac{e^{-\mu_a \cdot \frac{h}{x}}}{x} \cdot \epsilon(x)\,\mathrm{d}x. \end{aligned} \tag{7}$$